Topic 13: Properties

When you are simplifying a numerical expression, you may be asked to justify or explain your steps. In your justifications, it helps to refer to properties of rational numbers by name.

Problem 13.1

A. 1. Copy and complete the table.

Property	Algebra	Arithmetic
Associative	$(a + b) + c = a + (b + c)$ $(a \times b) \times c = a \times (b \times c)$	$(-7 + 3) + 9 = -7 + (3 + 9)$ $(5 \times \frac{1}{2}) \times 10 = 5 \times (\frac{1}{2} \times 10)$
Commutative	$a + b = b + a$ $a \times b = b \times a$	$-8.01 + (-12) = -12 + (-8.01)$ $5 \times 4 = 4 \times 15$
Identity	$a + 0 = 0 + a = a$ $1 \times a = a \times 1 = a$	$\frac{2}{3} + 0 = 0 + \frac{2}{3} = \frac{2}{3}$ $1 \times (-\frac{12}{15}) = (-\frac{12}{15}) \times 1 = -\frac{12}{15}$
Inverse	$a + (-a) = 0$ $\frac{1}{a} \times a = 1$	$34 + (-34) = 0$ $\frac{1}{18} \times 18 = 1$
Distributive	$a(b + c) = ab + ac$ $ab + ac = a(b + c)$	$-2 \times (6 + 5) = -2 \times 6 + (-2) \times 5$ $\frac{3}{4} \times 12 + \frac{3}{4} \times 16 = \frac{3}{4} \times (12 + 16)$

B. Name the property shown in each equation.

1. $5.81 + (-5.81) = 0$

2. $-4(x + 2) = -4x + (-8)$

3. $16\frac{1}{2} + 0 = 16\frac{1}{2}$

4. $\frac{3}{5} + \left(\frac{1}{5} + y\right) = \left(\frac{3}{5} + \frac{1}{5}\right) + y$

5. $0.68(-5) = -5(0.68)$

6. $23h - 46 = 23(h - 2)$

Sometimes you will need to use more than one property to simplify an expression. You can do the simplification in steps, using one property for each step.

Problem 13.2

A. The steps below show one way to simplify $\frac{1}{3} \cdot (3 \cdot 8) + 7 - 8$.

$\frac{1}{3} \cdot (8 \cdot 3) + 7 - 12$

$\frac{1}{3} \cdot (3 \cdot 8) + 7 - 12$

$\left(\frac{1}{3} \cdot 3\right) \cdot 8 + 7 - 12$

$1 \cdot 8 + 7 - 12$

$8 + 7 - 12$

$15 - 12$

3

Copy and complete the steps above, naming a property or operation to the right of each step.

B. Simplify each expression. Use a property or operation to justify each step.

1. $6 - 8x + 4(5 + 2x)$

2. $7.2a + 4(-3.5 + a)$

Exercises

Name the property illustrated in each equation.

1. $0.85 + (3.5 + 4.15) = (0.85 + 3.5) + 4.15$

2. $3d - 15 = 3(d - 5)$

3. $0 + (-1.6) + 2.4 = -1.6 + 2.4$

4. $\frac{1}{2} \times \frac{2}{1} \times \frac{1}{4} = 1 \times \frac{1}{4}$

5. $15(2c - 8) = 30c - 120$

6. $-3.2 + (-8.5x) = -8.5x + (-3.2)$

7. $123 + (-43) + 0 + (-15) = 123 + (-43) + (-15)$

Simplify each expression. Use a property or operation to justify each step.

8. $-4 + \frac{5}{2} + \frac{6}{5} + \frac{7}{2} + \frac{4}{5}$

9. $5m + 6 + 3(m + 2)$

10. $-2\left(\frac{1}{2}k + \frac{1}{3}\right) + 6 + \frac{2}{3}$

Topic 11: Properties

Mathematical Goals

- Identify properties of rational numbers
- Use properties of rational numbers to simplify expressions and justify steps in calculations

Teaching Guide

Some students may need to use Topic 10 to review number properties as an introduction to Topic 11.

Remind students that some properties are similar for addition and multiplication, such as the associative and commutative properties, while other properties, such as the inverse properties, may look different. Understanding the similarities between properties will help students to differentiate between different properties. For example, the associative properties involve associating different terms around an operation, while the inverse properties involve values that add (or multiply) to give the additive (or multiplicative) inverses.

Until students are comfortable with the properties, they should write out the complete names (Commutative Property of Addition, Commutative Property of Multiplication) to prevent confusion with other operations. For example, the commutative property is only true for addition and multiplication, but not for subtraction or division.

To summarize Problem 11.2A, ask:

- *What is the difference between a property and an operation?*
- *Why might you want to use a property in the first step rather than multiplying 3 and 8?*
- *How does writing each property or operation next to each step help you check the answer once you have simplified the expression?*

To summarize Problem 11.2B1, ask:

- *Which property would you use first?*
- *The first step in one student's solution is to use the Distributive Property on $4(5 + 2x)$. The first step in another student's solution is to use the Commutative Property of Addition on $5 + 2x$. Is one solution correct?*
- *Can you use the commutative property to write $8x - 6 + 4(5 + 2x)$ in a simpler form?*

Vocabulary

- Associative Property of Addition
- Associative Property of Multiplication
- Commutative Property of Addition
- Commutative Property of Multiplication
- Identity Property of Addition
- Identity Property of Multiplication
- Inverse Property of Addition
- Inverse Property of Multiplication
- Distributive Property

Assignment Guide for Topic 11

Core 1–10

Answers to Topic 11

Problem 11.1

A. See Figure 1.

B. **1.** Inverse Property of Addition

2. Distributive Property

3. Identity Property of Addition

4. Associative Property of Addition

5. Commulative Property of Multiplication

6. Distributive Property

Problem 11.2

A. **1.** $\frac{1}{3} \cdot (8 \cdot 3) + 7 - 12$

$\frac{1}{3} \cdot (3 \cdot 8) + 7 - 12$ (Comm. Prop.)

$\left(\frac{1}{3} \cdot 3\right) \cdot 8 + 7 - 12$ (Assoc. Prop.)

$1 \cdot 8 + 7 - 12$ (Inverse Prop.)

$8 + 7 - 12$ (Identity Prop.)

$15 - 12$ (addition)

3 (subtraction)

B. **1.** $6 - 8x + 4(5 + 2x)$

$6 - 8x + 20 + 8x$ (Distributive Prop.)

$6 - 8x + 8x + 20$ (Comm. Prop)

$6 + 0 + 20$ (Inverse Prop.)

$6 + 20$ (Identity Prop.)

26 (addition)

2. $7.2a + 4(-3.5 + a)$

$7.2a + (-14) + 4a$ (Distributive Prop.)

$7.2a + 4a + (-14)$ (Comm. Prop.)

$11.2a + (-14)$ (addition)

Exercises

1. Associative Property of Addition

2. Distributive Property

3. Identity Property of Addition

4. Inverse Property of Multiplication

5. Distributive Property

6. Commutative Property of Addition

7. Identity Property of Addition

8. $-4 + \frac{5}{2} + \frac{6}{5} + \frac{7}{2} + \frac{4}{5}$

$-4 + \frac{5}{2} + \frac{7}{2} + \frac{6}{5} + \frac{4}{5}$ (Comm. Prop.)

$-4 + \frac{12}{2} + \frac{10}{5}$ (addition)

$-4 + 6 + 2$ (division)

4 (addition)

9. $5m + 6 + 3(m + 2)$

$5m + 6 + 3m + 6$ (Distributive Prop.)

$5m + 3m + 6 + 6$ (Comm. Prop.)

$8m + 12$ (addition)

10. $-2\left(\frac{1}{2}k + \frac{1}{3}\right) + 6 + \frac{2}{3}$

$-2 \times \frac{1}{2}k + \left(-\frac{2}{3}\right) + 6 + \frac{2}{3}$ (Distributive Prop.)

$-k + \left(-\frac{2}{3}\right) + 6 + \frac{2}{3}$ (Identity Prop. of Mult.)

$-k + 6 + \left(-\frac{2}{3}\right) + \frac{2}{3}$ (Comm. Prop. of Add.)

$-k + 6 + 0$ (Inverse Prop.)

$-k + 6$ (Identity Prop. of Add.)

Figure 1

Property	Algebra	Arithmetic
Associative	$(a + b) + c = a + (b + c)$ $(a \times b) \times c = a \times (b \times c)$	$(-7 + 3) + 9 = -7 + (3 + 9)$ $(5 \times \frac{1}{2}) \times 10 = 5 \times (\frac{1}{2} \times 10)$
Commutative	$a + b = b + a$ $a \times b = b \times a$	$-8.01 + (-12) = -12 + (-8.01)$ $5 \times 4 = 4 \times 15$
Identity	$a + 0 = 0 + a = a$ $1 \times a = a \times 1 = a$	$\frac{2}{3} + 0 = 0 + \frac{2}{3} = \frac{2}{3}$ $1 \times \left(-\frac{12}{15}\right) = \left(-\frac{12}{15}\right) \times 1 = -\frac{12}{15}$
Inverse	$a + (-a) = 0$ $\frac{1}{a} \times a = 1$	$34 + (-34) = 0$ $\frac{1}{18} \times 18 = 1$
Distributive	$a(b + c) = ab + ac$ $ab + ac = a(b + c)$	$-2 \times (6 + 5) = -2 \times 6 + (-2) \times 5$ $\frac{3}{4} \times 12 + \frac{3}{4} \times 16 = \frac{3}{4} \times (12 + 16)$